Holomorphic function

Intro

Holomorphic function refers function that one or more complex parameter that is differentiable for complex number in the neighborhood of each point in a domain in complex coordinate space

Def

The definition of differentiable function on complex domain is still same in real (which I have learned before).

=

Property

Given holomorphic function and , scalar and . They always hold:

1. Linearity of differentiable function.

= while =

Thus =

1. Product rule in differentiable function.

=

1. Quotent rule in differentiable function.

=

1. Chain rule in differentiable function.

=

1. They are also holomorphic

(product of two functions)

( quotent of two functions) (it requires that )

(composition of two functions)

Proof

1. Linearity:

Given a point at interval which defined above and on the function .

=

=

(by letting , it is the 2th definition of derivatives.)

=> =

=

=

=

=

=

= +

2.

Consider the following limt.

=

=

=

=

\*+ \*

=

=

=

=

3. For Quotent rule, just take recipocal of then differentiation it with chain rule , finally simplify it.

4. Simple. Skip.

5. Holomorphic function

is also holomorphic since the property of sum of differentiation. (See 1th point in property section)

And is also , just replace to .

is holomorphic, see the proof of product rule.

And is also

is also holomorphic, see the proof of the chain rule.

Ref

[Holomorphic function - Wikipedia](https://en.wikipedia.org/wiki/Holomorphic_function)